International Journal on Emerging Technologies **10**(2b): 54-56(2019)

ISSN No. (Print): 0975-8364
ISSN No. (Online): 2249-3255

New Entropic Modeling for Discrete Probability Distributions

Ruchi Handa[1, 2*], Rajesh Kumar Narula[3] and C.P. Gandhi[4]

[1]Research Scholar, IK Gujral Punjab Technical University, Kapurthala, India.
[2]Department of Applied Sciences, Khalsa College of Engineering and Technology, Amritsar, India.
[3] Department of Mathematical Sciences, IK Gujral Punjab Technical University, Kapurthala, India.
[4]Department of Mathematics, University School of Sciences, Rayat Bahra University, Kharar, Mohali, India.

(Corresponding author: Ruchi Handa)
(Received 02 May 2019, Revised 10 July 2019, Accepted 08 August 2019)
(Published by Research Trend, Website: www.researchtrend.net)

ABSTRACT: It is the known fact that as the probabilistic measures of entropy is important mathematical tools in many optimization problems because of the importance of occurring events. In the present communication, we introduce new generalized information model based upon discrete probability distributions and we further discuss some of its properties.

Keywords: Entropy, Probability distribution, Non-negativity, Concavity, Increasing function.

I. INTRODUCTION

The information entropy introduced by Shannon [9] measures the amount of uncertainty contained in a probabilistic experiment, and is not a single monolithic concept. It can appear in several guises. It can arise in probabilistic phenomenon type. On the other hand, it can also appear in a deterministic phenomenon where we know that the outcome is not a chance event, but we are fuzzy about the possibility of the specific outcome. This type of uncertainty arising out of fuzziness is the subject of investigation of the relatively new discipline of fuzzy set theory. In this paper our objective is to deal with the probabilistic measures of uncertainty called entropy given by

$$H(P) = -\sum_{i=1}^{n} p_i \operatorname{In} p_i \qquad (1.1)$$

Some other entropic models are:

$$H^{\alpha}(P) = \frac{1}{1-\alpha}\left(\sum_{i=1}^{n} p_i^{\alpha} - 1\right), \alpha \neq 1, \alpha > 0 \qquad (1.2)$$

Which is modified form of Havrada and Charvat [4] measure of entropy. Other entropic models are given by Wang [12], Chen and Geman [2], Parkash, Thukral and Gandhi [8], Sharma and Mittal [10], Vinocha and Hemlata [11], Chen [1], Cincotta and Giordano [3], Khozani and Bonakdari [5] etc. Parkash and Mukesh [7] applied their entropic models to queuing theory whereas Parkash and Kakkar [6] developed some new measures of information so as to apply their findings in coding theory. In section 2, we discuss the study of a new generalized entropic model for discrete distribution. In section 3, we discuss about the conclusion.

II. A NEW PROBABILISTIC MEASURE OF ENTROPY FOR DISCRETE PROBABILITY DISTRIBUTIONS

The objective of this paper is to discuss the concavity of the entropic model of order α given by

$$H^{\alpha}(P) = \frac{1}{2^{1-\alpha}-1}\left[\sum_{i=1}^{n} p_i^{\alpha} - \left\{\sum_{i=1}^{n} p_i^{\frac{1}{\alpha}}\right\}^{\alpha}\right]; \alpha \neq 1, \alpha > 1 \qquad (2.1)$$

We have

$$\lim_{\alpha \to 1} H^{\alpha}(P) = -2\sum_{i=1}^{n} p_i \log p_i$$

which is Shannon's [9] entropy except for a multiple constant implying that $H^{\alpha}(P)$ is a generalized entropy model.

Next, to make $H^{\alpha}(P)$ authentic, we study its properties as follows:

1. Non-Negativity

For $\alpha > 1$, we have $p_i^{\alpha} \leq p_i^{\frac{1}{\alpha}}$

$$\Rightarrow \frac{1}{2^{1-\alpha}-1}\sum_{i=1}^{n} p_i^{\alpha} - \left(\sum_{i=1}^{n} p_i^{\frac{1}{\alpha}}\right)^{\alpha} \geq 0$$

$$\Rightarrow H^{\alpha}(P) \geq 0$$

2. **Symmetry**: $H^{\alpha}(P)$ is permutationally symmetric because of the fact that it remains invariant if $p_1, p_2, p_3, ..., p_n$ are rearranged among themselves.

3. Continuity: $H^{\alpha}(P)$ is a continuous function of p_i for all p_i's.

4. **Concavity:** To prove concavity property, we proceed as follows:

We have

$$\frac{\partial}{\partial p_i} H^{\alpha}(P) = \frac{1}{2^{1-\alpha}-1}\left[\alpha p_i^{\alpha-1} - p_i^{\frac{1}{\alpha}-1}\left\{\sum_{i=1}^{n} p_i^{\frac{1}{\alpha}}\right\}^{\alpha-1}\right]$$

Also

$$\frac{\partial^2}{\partial p_i^2}H^\alpha(P)=\frac{1}{2^{1-\alpha}-1}\left[\alpha(\alpha-1)\,p_i^{\alpha-2}+\frac{(\alpha-1)}{\alpha}\left\{\sum_{i=1}^{n}p_i^{\frac{1}{\alpha}}\right\}^{\alpha-1}\left(p_i^{\frac{1}{\alpha}-2}-\frac{p_i^{\frac{2}{\alpha}-2}}{\sum_{i=1}^{n}p_i^{\frac{1}{\alpha}}}\right)\right]$$

$$=\frac{\alpha-1}{2^{1-\alpha}-1}\left[\alpha p_i^{\alpha-2}+\frac{1}{\alpha}\left\{\sum_{i=1}^{n}p_i^{\frac{1}{\alpha}}\right\}^{\alpha-1}p_i^{\frac{1}{\alpha}-2}\left(1-\frac{p_i^{\frac{1}{\alpha}}}{\sum_{i=1}^{n}p_i^{\frac{1}{\alpha}}}\right)\right]<0\ as\ \left(1-\frac{p_i^{\frac{1}{\alpha}}}{\sum_{i=1}^{n}p_i^{\frac{1}{\alpha}}}\right)>0$$

Thus, $H^\alpha(P)$ is concave. Moreover, the graphical presentation of $H^\alpha(P)$ provided in Fig. 1 obtained numerically from Table 1 proves the concavity of the measure (2.1).

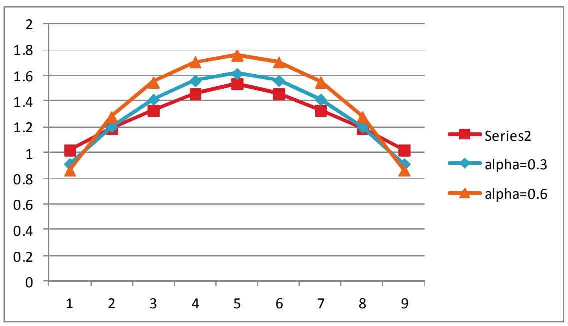

Fig. 1. Concavity of $H^\alpha(P)$ with respect to P.

Under the above properties, we assert that the proposed model is a legitimate entropic model.

Next, we learn some more desirable properties of $H^\alpha(P)$.

5. Expansibility: We have $H^\alpha(p_1,p_2,p_3,...,p_n,0)=H^\alpha(p_1,p_2,p_3,...,p_n)$ which shows that the $H^\alpha(P)$ is invariant upon addition of an impractical event.

6. Uncertainty for Degenerate Distributions: For n degenerate probability distributions $\{(1,0,...,0),(0,1,...,0),...,(0,0,...,1)\}$, we have $H^\alpha(P)=0$ which provides the indication that certain outcomes always result in zero uncertainty.

7. Entropy Maximization: In many real life situations, we usually come across many optimization problems dealing with different disciplines. One such a problem is to find the maximum value of entropy function so as to reduce uncertainty contained in a probabilistic experiment. To find the extreme value of the proposed entropy, we use Lagrange's method of maximum multipliers and thus our problem reduces in maximizing $H^\alpha(P)$ under the likely restriction $\sum_{i=1}^{n}p_i=1$.

To apply the method, we have

$$L\equiv\frac{1}{2^{1-\alpha}-1}\left[\sum_{i=1}^{n}p_i^\alpha-\left\{\sum_{i=1}^{n}p_i^{\frac{1}{\alpha}}\right\}^\alpha\right]-\lambda\left(\sum_{i=1}^{n}p_i-1\right)\qquad(2.2)$$

Taking $\frac{\partial f}{\partial p_1}=\frac{\partial f}{\partial p_2}=\ldots=\frac{\partial f}{\partial p_n}=0$, we get

$$\frac{1}{2^{1-\alpha}-1}\left[\alpha p_1^{\alpha-1}-1\right]-\lambda=\frac{1}{2^{1-\alpha}-1}\left[\alpha p_2^{\alpha-1}-1\right]-\lambda=...=\frac{1}{2^{1-\alpha}-1}\left[\alpha p_n^{\alpha-1}-1\right]-\lambda$$

which is promising only if $p_1=p_2=...=p_n$. Moreover, the natural constraint $\sum_{i=1}^{n}p_i=1$ gives

$$p_1=p_2=...=p_n=\frac{1}{n}$$

Hence, we conclude that $H^\alpha(P)$ possesses maximum value when $p_1=p_2=...=p_n=\frac{1}{n}$. This most wanted property coveys that the entropic model attains its maxima at the uniform distribution.

8. **Maximum Value:** The maximum value of the entropy is given by:

$$\left[H^\alpha(P)\right]_{\max}=\frac{1}{2^{1-\alpha}-1}\left[\sum_{i=1}^{n}\left(\frac{1}{n}\right)^\alpha-\left\{\sum_{i=1}^{n}\left(\frac{1}{n}\right)^{\frac{1}{\alpha}}\right\}^\alpha\right];\alpha\neq1,\ \alpha>0$$

Thus, we have

$$\left[H^\alpha(P)\right]_{\max}=\frac{1}{2^{1-\alpha}-1}\left[1-\left\{n^{\alpha-1}\right\}\right]>0;\ \alpha\neq1,\ \alpha>0$$

which shows that $H^\alpha(P)$ increases for the large values of *n*, which again delivers a most attractive result because of the fact that the maximum entropy function should always have one direction, that is, the direction of increase.

III. CONCLUSION

We found that the generalized measures of information induce flexibility and unbiasedness into the system and hence find their significance and importance towards application areas in different disciplines. Further, we made investigations and new proposal for generalized parametric entropic models. Such parametric as well as non-parametric mathematical models can be developed so that their applications may be provided.

CONFLICT OF INTEREST

Author have no any conflict of interest.

Table 1: Numerical numbers for $H^{\alpha}(P)$ provided in Fig. 1.

α	p_i	$0\cdot1$	$0\cdot2$	$0\cdot3$	$0\cdot4$	$0\cdot5$	$0\cdot6$	$0\cdot7$	$0\cdot8$	$0\cdot9$
$0\cdot1$	$H^{\alpha}(P)$	1.02053	1.18844	1.32959	1.45671	1.53588	1.45671	1.32959	1.18844	1.02053
$0\cdot3$	$H^{\alpha}(P)$	0.91255	1.20083	1.41417	1.56071	1.61557	1.56071	1.41417	1.20083	0.91255
$0\cdot6$	$H^{\alpha}(P)$	0.86424	1.27916	1.54959	1.70627	1.75785	1.70627	1.54959	1.27916	0.86424

REFERENCES

[1]. Chen, Y. (2006). Properties of quasi-entropy and their applications. *Journal of Southeast University Natural Sciences,* **36**, 222-225.

[2]. Chen, T. L., & Geman, S. (2008). On the minimum entropy of a mixture of unimodal and symmetric distributions. *IEEE Transactions on Information Theory,* **54**, 3166-3174.

[3]. Cincotta, P. M., & Giordano, C. M. (2018). Phase correlations in chaotic dynamics: a Shannon entropy measure. *Celestial Mech. Dynam. Astronom.*, **130**(11), 130:74.

[4]. Havrada, J. H., & Charvat, F. (1967). Quantification methods of classification process: Concept of structural α-entropy, *Kybernetika,* **3**, 30-35.

[5]. Khozani, Z. S., & Bonakdari, H. (2018). Formulating the shear stress distribution in circular open channels based on the Renyi entropy. *Phys. A*, **490**, 114–126.

[6]. Parkash, O., & Kakkar, P. (2014). New measures of information and their applications in coding theory. *Canadian Journal of Pure and Applied Sciences,* **8**(2), 2905-2912.

[7]. Parkash, O., & Mukesh. (2015). Contribution of maximum entropy principle in the field of queuing theory. *Communications in Statistics-Theory and Methods,* **45**(12), 3464-3472.

[8]. Parkash, O., Thukral, A.K., & Gandhi, C.P. (2011). Some new information measures based upon sampling distributions. *International Journal of Engineering and Applied Sciences*, **7**(2), 75-79.

[9]. Shannon, C. E. (1948). A mathematical theory of communication. *Bell. Sys. Tech. Jr.,* **27**, 379-423, 623-659.

[10]. Sharma, B. D., & Mittal, D. P. (1975). New non-additive measures of entropy for a discrete probability distributions. *Journal of Mathematical Sciences,* **10**, 122-133.

[11]. Vinocha, O. P., & Hemlata. (2004). Havrada-Charvat's entropy and the probability of error. *Journal of Rajasthan Academy of Physical Sciences,* **3**: 101-110.

[12]. Wang, Q. A. (2008). Probability distribution and entropy as a measure of uncertainty. *Journal of Physics A: Mathematical and Theoretical,* **41**, 1-8.